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HEAT EXCHANGE BETWEEN AN INFILTRATED GRANULAR BED AND A SURFACE

Yu. S. Teplitskii

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Comparative analysis of the existing relations for calculation of convective heat exchange between an infiltrated granular bed and surfaces of different geometry has been carried out. It is shown that different characteristic scales of the process of heat and mass transfer correspond to laminar and turbulent modes of filtrational flow. The occurrence of an analogy with heat transfer of a single-phase medium in a laminar mode has been found. A correlation for calculation of heat exchange between a vertical cylinder and an infiltrated granular bed has been obtained.

Convective heat exchange between macrosurfaces and an infiltrated (blown-through) granular bed occurs in different power plants and technological equipment. Comprehensive knowledge of the laws governing heat transfer within a wide range of variation of determining parameters is required to provide necessary reliability of design of such apparatuses.

The present paper is aimed at investigation of relations between heat transfer and a hydrodynamic mode of filtrational flow in a granular bed through analysis of characteristic scales of the process and, on this basis, at determination of physically justified limits of applicability of the available relations to calculation of heat exchange between a granular bed and a surface. We consider most important, from the point of view of practice, variants of heat transfer: in longitudinal flow past a plate (direction of gas/liquid filtration relative to the surface is implied); in longitudinal flow past a cylinder; and in transverse flow past a cylinder with the walls filled with a granular bed.

Heat Transfer in Longitudinal Flow past a Plate. In [1], the known Boussinesque solution

$$\alpha = \left(\frac{4c_{\mathrm{f}}\,\rho_{\mathrm{f}} u\lambda}{\pi L}\right)^{0.5} . \qquad (1)$$

was used for calculation of the plate-length-mean coefficient of heat transfer. Formula (1) corresponds to a simplest one-zone model, which considers a granular bed as quasihomogeneous medium with thermophysical characteristics constant throughout the volume. In [1] it was shown that (1) satisfactorily describes experimental data in beds of particles with d = 0.3–0.8 mm at Re_d = 2–30. In [2], based on a more physical two-zone model of the granular bed, which allows for the presence of the zone of elevated porosity near the heat-transfer surface, the computational relation

$$\alpha = \frac{2\lambda_{\mathrm{eff}}}{D_{\mathrm{a}}}\,\frac{\lambda^0 s - f^2\xi_d}{1 + \lambda^0 s\xi_d} , \qquad (2)$$

which describes heat transfer at Re_d = 2–2600, was obtained. It can be easily seen that for $\xi_d \to 0$ (one-zone model) formula (2) takes on the form

$$\alpha = (c_{\mathrm{f}}\rho_{\mathrm{f}} u\lambda / L)^{0.5} . \qquad (3)$$

Expression (3) coincides with (1) with an accuracy of the coefficient $(4/\pi)^{0.5} \approx 1.13$. The study of the asymptotic behavior of (2) for $\xi_d \to 0$ showed that, beginning with $d/D_{\mathrm{a}} \approx 5\cdot10^{-3}$, formulas (2) and (3) virtually [with an error not higher than 5% (usual for measurements of the heat-transfer coefficient)] coincide.

A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, 15 P. Brovka Str., Minsk, 220072, Belarus; email: dsl@hmti.ac.by. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 76, No. 6, pp. 151–155, November–December, 2003. Original article submitted March 12, 2003.

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Heat Transfer in Longitudinal Flow past a Cylinder. Gabor [1] obtained a simple relation similar to (1):

$$\alpha = \left(\frac{4c_f\,\rho_f u\lambda}{\pi L}\right)^{0.5} + \lambda/D\,, \tag{4}$$

which was verified for Re_d = 2–213. In [2], within the framework of the already-mentioned two-zone model the following was obtained for α:

$$\alpha = \frac{2\lambda_{\mathrm{eff}}}{D_a}\,\frac{\lambda^0 s + f^2\xi_d/K^*}{1 + \lambda^0 s\xi_d/K^*}\,K^*\,. \tag{5}$$

This formula was checked within a wide range of Reynolds numbers Re_d = 2–2600. It is easy to see that at $K^* = 1$ (plane surface) (5) changes over to (2). When $\xi_d \to 0$, (5) takes on the form

$$\alpha = K^*\,(c_f\rho_f u\lambda/L)^{0.5}\,. \tag{6}$$

As for the plane case, the calculations by (6) are in good agreement with formula (4).

Based on (5), which reflects the special features of heat transfer in the granular bed most adequately, we can write a general form of the functional dependence of the mean coefficient of heat exchange of the granular bed with the vertical cylinder of height L:

$$\mathrm{Nu}_L = f\left(\mathrm{Re}_L,\,\mathrm{Pr},\,\frac{\lambda_s}{\lambda_f},\,\varepsilon,\,\frac{d}{D_a},\,\frac{d}{L},\,\frac{d}{D}\right). \tag{7}$$

Despite the large number of determining dimensionless parameters, the hydrodynamic mode of flow in the granular bed is determined only by the specific form of Nu_L as a function of Re_L. Consequently, the study of characteristic scales of the process of heat transfer, which correspond to one flow mode or another, does not require analysis of the equation which involves d/D_a and d/D (they reflect the value of the near-wall resistance and curvature of the heat-transfer surface). Therefore, it suffices to restrict ourselves to investigation of Eq. (1) or Eq. (3). Equation (3) can be given a dimensionless form:

$$\mathrm{Nu}_L = \left(\frac{\lambda}{\lambda_f}\right)^{0.5}\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{0.5}\,. \tag{8}$$

It is easy to see that (8) is a partial case of the general expression (7) if we take into account that the quantity λ/λ_f is given by the expression [3]

$$\frac{\lambda}{\lambda_f} = \frac{\lambda_s^0}{\lambda_f} + 0.1\,\mathrm{Re}_d\,\mathrm{Pr}\,, \tag{9}$$

where

$$\frac{\lambda_s^0}{\lambda_f} = 1 + \frac{(1-\varepsilon)\,(1-\lambda_f/\lambda_s)}{\lambda_f/\lambda_s + 0.28\varepsilon^{0.63(\lambda_s/\lambda_f)^{0.18}}}\,. \tag{10}$$

With account for (9) we represent expression (8) as

$$\mathrm{Nu}_L = \left(\frac{\lambda_s^0}{\lambda_f} + 0.1\,\mathrm{Re}_d\,\mathrm{Pr}\right)^{0.5}\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{0.5}\,. \tag{11}$$

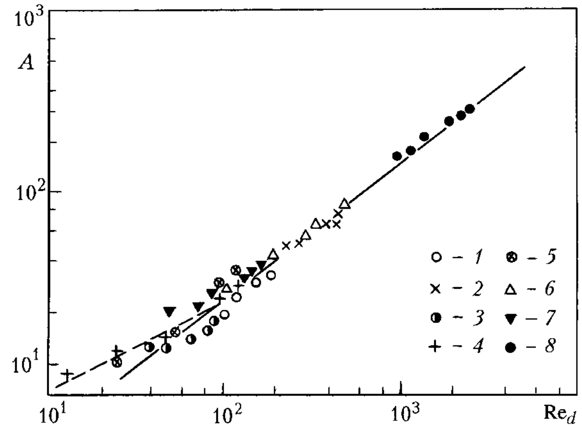

Fig. 1. Generalization of experimental data on heat exchange between a vertical cylinder and an infiltrated granular bed: 1–3) [2] (d = 3.0, 5.7, 1.75 mm); 4–7) [5] (d = 0.62, 1.02, 2.37, 1.02 mm); 8) [6] (d = 3.1 mm); fluidizing agent: 7) CO_2; the rest) air; pressure: 1–3) atmospheric; 4–7) p = 0.1–0.93 MN/m^2; 8) p = 0.68–8.44 MN/m^2; solid line — according to Eq. (14). $A \equiv \mathrm{Nu}_d \Big/ \left(\mathrm{Pr}^{0.4}\left(\frac{d}{L}\right)^{0.33}\left(\frac{d}{D}\right)^{0.2}\left(\frac{\lambda_s}{\lambda_f}\right)^{0.07}\right)$.

We consider two limiting cases at Pr = 0.7 and λ_s^0/λ_f = 3–6.

1. *Small* Re_d. When $\mathrm{Re}_d \le 5$, $\lambda/\lambda_f \approx \lambda_s^0/\lambda_f$ and formula (11) is

$$\mathrm{Nu}_L = \left(\frac{\lambda_s^0}{\lambda_f}\right)^{0.5} \mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{0.5}\,. \tag{12}$$

The coefficient λ_s^0/λ_f does not depend on Re_d and (12) has the typical form of the formula for calculation of heat transfer of a one-phase medium in a laminar mode. Consequently, (12) describes heat transfer in the granular bed in a laminar mode when viscosity forces prevail and the Darcy law holds [4]. In this mode, interaction between individual jets is virtually absent and the process is determined by the only characteristic scale — the surface length L.

2. *Large* Re_d. When $\mathrm{Re}_d \ge 80$, $\lambda/\lambda_f \approx 0.1\mathrm{Re}_d\mathrm{Pr}$ and formula (11) takes on the form

$$\mathrm{Nu}_L = 0.316\,\mathrm{Re}_L\,\mathrm{Pr}\left(\frac{d}{L}\right)^{0.5}. \tag{13}$$

Dependence (13) corresponds to a turbulent flow mode when effective heat transfer, which is caused by mixing of individual gas jets, formation of vortices in shadow regions behind particles, and interaction of these vortices with each other, becomes substantial. The scale of these vortices is d, which results in the appearance of a new characteristic dimension in (13).

Formula (5) is not quite illustrative and is not convenient for practical calculations. Therefore, using (7) as a basis, we made an exponential approximation of it for a turbulent mode:

$$\mathrm{Nu}_L = 0.68\,\mathrm{Re}_L^{0.77}\,\mathrm{Pr}^{0.4}\left(\frac{d}{L}\right)^{0.1}\left(\frac{d}{D}\right)^{0.2}\left(\frac{\lambda_s}{\lambda_f}\right)^{0.07}. \tag{14}$$

Experimental data of [2, 5, 6] together with results of the calculation by (14) are given in Fig. 1 [for convenience, formula (14) is presented in terms of $\mathrm{Nu}_d \to \mathrm{Re}_d$]. As is seen, the formula obtained describes experimental results within the range Re_d = 40–2600 well (the root-mean-square deviation of experimental points from those calculated by

(14) is 7%). When $\mathrm{Re}_d < 40$, the dependence on Re_d is weaker and close to $\mathrm{Re}_d^{0.5}$ (dashed line), which is in full conformity with formula (12) describing heat transfer in a laminar mode. Thus, a noticeable effect of heat transfer due to vortex formation and mixing of neighboring jets (turbulent flow mode) begins approximately at $\mathrm{Re}_d = 40$.

Heat Transfer in Transverse Flow past a Cylinder. In [7], the dependence

$$\mathrm{Nu}_D = 3.4\,\mathrm{Re}_D^{0.5}\,\mathrm{Pr}^{0.33}\,, \tag{15}$$

which describes experimental data within the range $\mathrm{Re}_D = 15\text{–}100$ and $\mathrm{Re}_d \leq 40$, is obtained for a laminar flow mode. It was emphasized in [7] that a characteristic scale — cylinder diameter — corresponds to this model. At higher velocities, when effective heat transfer becomes substantial, the dependence of the heat-transfer coefficient on the rate of filtration changes and, what is most important, the characteristic scale of the process also changes — the particle diameter becomes the characteristic scale. In this case, the computational equation has the form [7]

$$\mathrm{Nu}_d = 0.31\,\mathrm{Re}_d^{0.8}\,\mathrm{Pr}^{0.4}\,. \tag{16}$$

The range of verification of (16) is $\mathrm{Re}_d = 40\text{–}4000$. We note that the lower value of $\mathrm{Re}_d = 40$ coincides with the limiting value of Re_d in the case of a vertical cylinder.

Heat Exchange with the Wall of a Round Tube Filled with a Granular Bed. In accordance with the classification of flow modes adopted in [8]: inertial mode ($5 \leq \mathrm{Re}_d \leq 80$), transient mode ($80 \leq \mathrm{Re}_d \leq 120$), and turbulent mode ($\mathrm{Re}_d > 120$), Dekhtyar' et al. obtained the following dependences for the case of the boundary condition of the second kind:

inertial mode

$$\mathrm{Nu}_{D_\mathrm{a}} = 7.5\,\mathrm{Re}_\mathrm{e}^{0.5}\,\mathrm{Pr}^{0.5}\,, \tag{17}$$

turbulent mode

$$\mathrm{Nu}_d = 0.4\,\mathrm{Re}_d^{0.67}\,\mathrm{Pr}^{0.4}\,. \tag{18}$$

Comparison of (17) and (18) shows that different characteristic scales correspond to different flow modes: D_a and d (with account for $\mathrm{Re}_d \approx \mathrm{Re}_\mathrm{e}$) correspond to the inertial mode and d corresponds to the turbulent mode.

It should be noted that appearance of the particle diameter among the characteristic dimensions for a laminar mode is caused by the fact that formula (17) generalizes the experimental data obtained, as was mentioned in [8], under the conditions where the contributions of thermal resistances of the near-wall zone and the zone core were commensurable. Consequently, the presence of the particle diameter in (17) reflects the effect of the near-wall zone on heat transfer.

As is seen, the laws governing a laminar mode can be realized in the infiltrated granular bed up to $\mathrm{Re}_d = 100$. The difference in specific values of the upper limit of the range of Reynolds numbers (see above) is caused by a certain arbitrariness of this quantity. Therefore, it is of importance to estimate its value by an independent technique — using the known Ergun formula [9], which describes the pressure difference per height unit of the granular bed

$$\frac{\Delta p}{L} = 150 \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3}\,\frac{\eta_\mathrm{f} u}{d^2} + 1.75 \cdot \frac{1-\varepsilon}{\varepsilon^3}\,\frac{\rho_\mathrm{f} u^2}{d}\,, \tag{19}$$

or in dimensionless form

$$\frac{\Delta p d^3 \rho_\mathrm{f}}{L \eta_\mathrm{f}^2} = 150 \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3}\,\mathrm{Re}_d + 1.75 \cdot \frac{1-\varepsilon}{\varepsilon^3}\,\mathrm{Re}_d^2\,. \tag{20}$$

We determine the number Re_d^* at which the linear and square terms in (20) are equal:

$$\mathrm{Re}_d^* = \frac{150\,(1-\varepsilon)}{1.75}\,. \tag{21}$$

For $\varepsilon = 0.4$ we obtain

$$\mathrm{Re}_d^* = 50\,. \tag{22}$$

It is seen from the examples given above that in the laminar mode of filtration under conditions where the contribution of the near-wall zone to total thermal resistance can be neglected, the linear size of the heat-transfer surface is, as in the case of a one-phase medium, the characteristic scale of the heat-transfer process. Since, under the conditions mentioned, the laws governing heat transfer in granular beds and in a one-phase medium are described by similar dependences (c.f., e.g., (12) and (15) with the known formulas for a one-phase medium [10]), we can speak about the analogy between these processes.

As a result, we show that different characteristic scales of the heat-transfer process correspond to different hydrodynamic modes of flow in the granular bed. Inertia forces, the effect of which becomes substantial with $\mathrm{Re}_d \approx 50$, are responsible for the appearance of effective heat transfer.

The occurrence of an analogy between the processes of heat transfer in a granular bed and in a one-phase medium in a laminar mode is found under conditions where the effect of the near-wall zone can be disregarded.

NOTATION

c, specific heat capacity, J/(kg·deg); d, particle diameter, m; d_e, hydraulic diameter of particles, m; D, diameter of the cylinder; D_a, diameter of the granular bed (apparatus), m; $f = \sqrt{\mathrm{Pe}\lambda^0/\varepsilon}$; K_0 and K_1, modified Bessell functions of the first kind and zero and first order; $K^* = K_1(s\xi^*)/K_0(s\xi^*)$; L, length of the surface, height of the cylinder, m; $\mathrm{Nu}_d = \alpha d/\lambda_f$, $\mathrm{Nu}_L = \alpha L/\lambda_f$, $\mathrm{Nu}_D = \alpha D/\lambda_f$, $\mathrm{Nu}_{D_a} = \alpha D_a/\lambda_f$, Nusselt numbers; $\mathrm{Pr} = c_f\eta_f/\lambda_f$, Prandtl number; $\mathrm{Pe} = c_f\rho_f u D_a^2/4\lambda L$, Peclet number; p, pressure, N/m^2; Δp, pressure difference, N/m^2; $\mathrm{Re}_d = ud\rho_f/\eta_f$, $\mathrm{Re}_D = uD\rho_f/\eta_f$, $\mathrm{Re}_L = uL\rho_f/\eta_f$, $\mathrm{Re}_e = ud_e\rho_f/\eta_f$, Reynolds numbers; $s = \sqrt{\mathrm{Pe}}$; u, rate of gas filtration, m/sec; α, heat-transfer coefficient, W/(m^2·deg); ε, porosity; η_f, dynamic viscosity of gas, N·sec/m^2; λ_s, thermal conductivity of material particles, W/(m·deg); λ, thermal conductivity of the granular bed, W/(m·deg); λ_f, molecular thermal conductivity of gas, W/(m·deg); $\lambda_{eff} = \lambda_f + 0.0061\rho_f c_f ud/\varepsilon$, effective thermal conductivity of a gas film near the heat-transfer surface, W/(m·deg); $\lambda^0 = \lambda/\lambda_{eff}$, $\xi^* = \xi_d + \xi_a$, $\xi_d = 0.2d/D_a$; $\xi_a = D/D_a$; ρ, density, kg/m^3. Subscripts: a, apparatus; e, equivalent; f, gas; s, particles; eff, effective.

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